Quantum computing and feedback

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(most of the slides are due to Mazyar Mirrahimi from QUANTIC)

Quantum information and technology

- Quantum algorithm: Fourier transform, factorization/discrete-log, quantum search algorithm, ….
- Quantum sensor: atomic clock, gradio/magneto meters, …
- Quantum network: key distribution, repeaters, ….
- Quantum simulator: solving physical many-body problems, simulators based on trapped-ions/ultracold-atom/superconducting-qubits , …
- Focus on **Quantum computing: qubit, error correction, noisy intermediate scale quantum (NISQ) computer, ….**

Preskill, J. (2023). Quantum computing 40 years later. In *Feynman lectures on computation* (pp. 193-244). CRC Press.

CONTEXT

Focus on the recent progress towards realization of quantum processors

Exploding general interest:

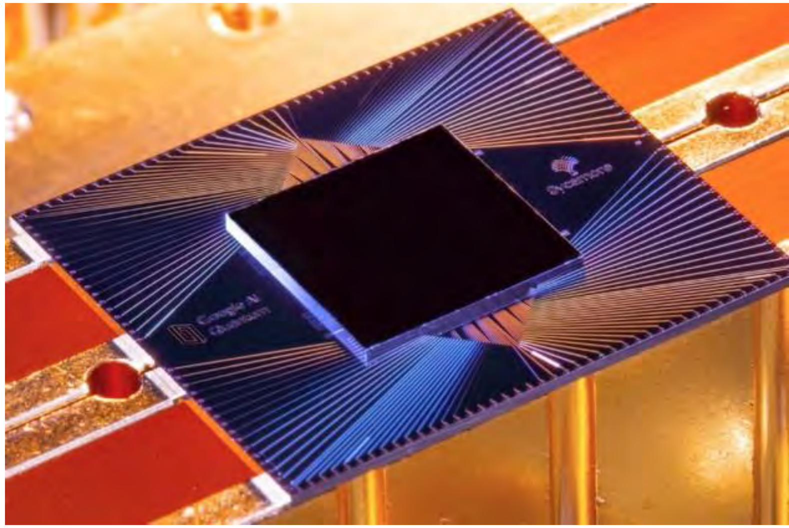
- Google: quantum supremacy with a superconducting quantum processor

  Gigantic non-uniform dice: in principle, sampling requires tens of years with conventional classical computers.

- Many other giants and startups: IBM, Amazon, Microsoft, Intel, ….
- In France: Alice&Bob, Pasqal, Quandela, Eviden, …
- Government initiatives: USA, Europe, China, …

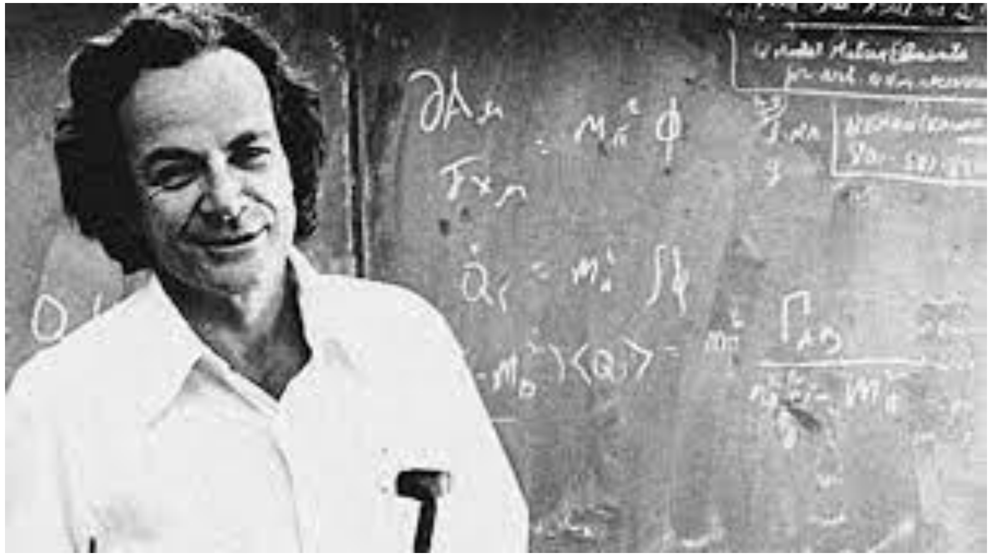
- Academic level: new field of quantum engineering combining expertise from Physics (theoretical and experimental), Applied Mathematics, and Computer Science.

  - New quantum institutes at many universities
  - New Master programs around the world



Google's Sycamore/Willow quantum processor

QUANTUM COMPUTING: how did it all start



Richard Feynman: in an invited talk in 1981

``Can quantum systems be probabilistically simulated by a classical computer? ''

His own response:

- *Quantum mechanics can't seem to be imitable by a local classical computer*
- *If you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy.*
- *Can you do it with a new kind of computer – a quantum computer? It's not a Turing machine, but a machine of a different kind.*

PROMISE OF QUANTUM COMPUTING

Theoretical construction:

- David Deutsch, 1985: Quantum Turing machine, and first surprising algorithms
- Ethan Bernstein and Umesh Vazirani, 1993: Universal and efficient quantum Turing machine, Exponential speedup, but for useless problems.

Algorithms:

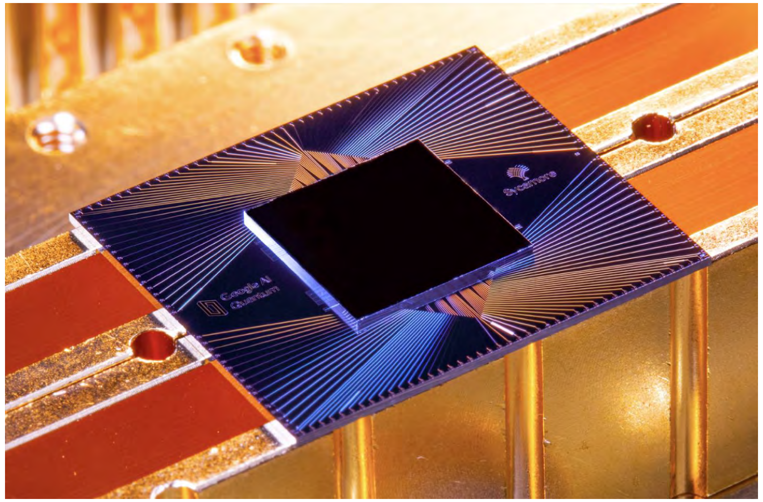
- Peter Shor, 1994: Factorizing large numbers, threat against most cryptography techniques
- Seth Lloyd, 1996: local Hamiltonian simulations
- Harrow, Hassim, Lloyd, 2009: Sometimes exponentially faster solution of systems of linear equations.
- …

David Deutsch

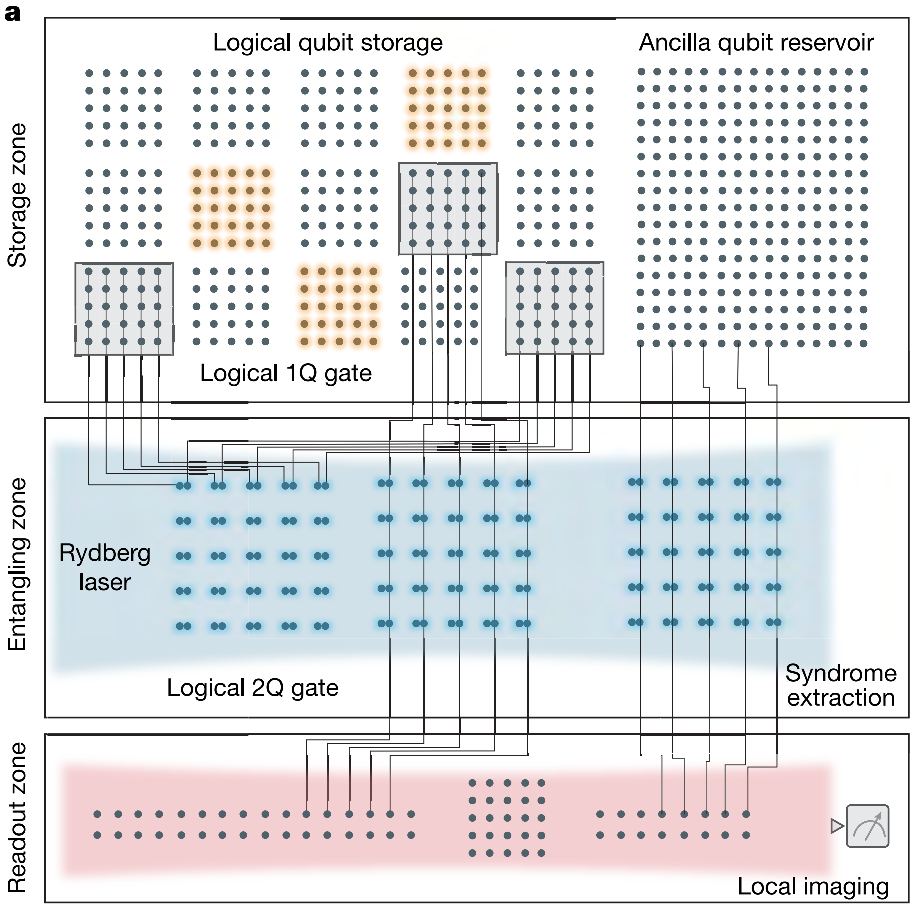
Umesh Vazirani

Peter Shor

QUANTUM BITS : PHYSICAL PLATFORMS

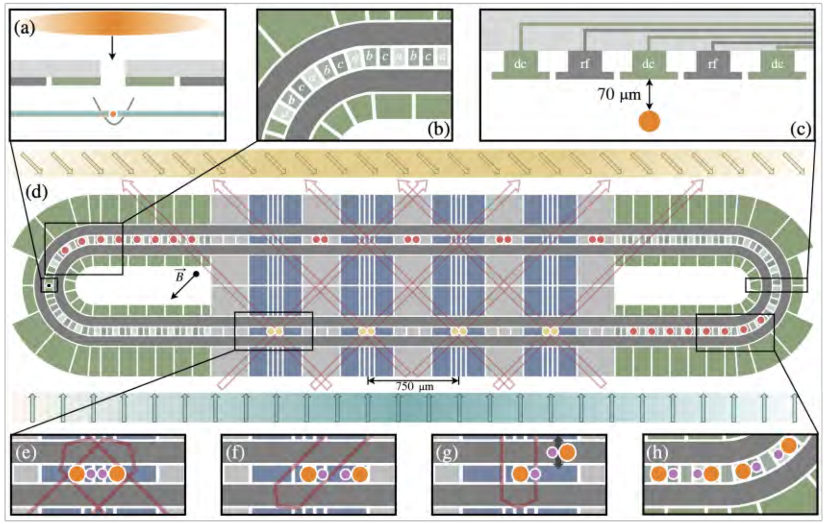


Superconducting circuits
(Google, IBM, Amazon, Alice&Bob, NordQuantique, QCI, …)



Neutral atoms in optical lattices
(PASQAL, QUERA, …)

Race-track trapped Ions



Quantinuum, PRX 2023

Trapped Ions
(Quantinuum, Ion-Q, …)

And a few other contenders…

Some of the most advanced platforms in the roadmap towards fault-tolerant quantum computation

QUANTUM versus CLASSICAL

Feature 1: Schrödinger equation replaces Newton's laws

State of a quantum system: Wave-function $|\psi\rangle \in \mathcal{H}$ with $\mathcal{H}$ a complex Hilbert space, for example $\mathbb{C}^n$.

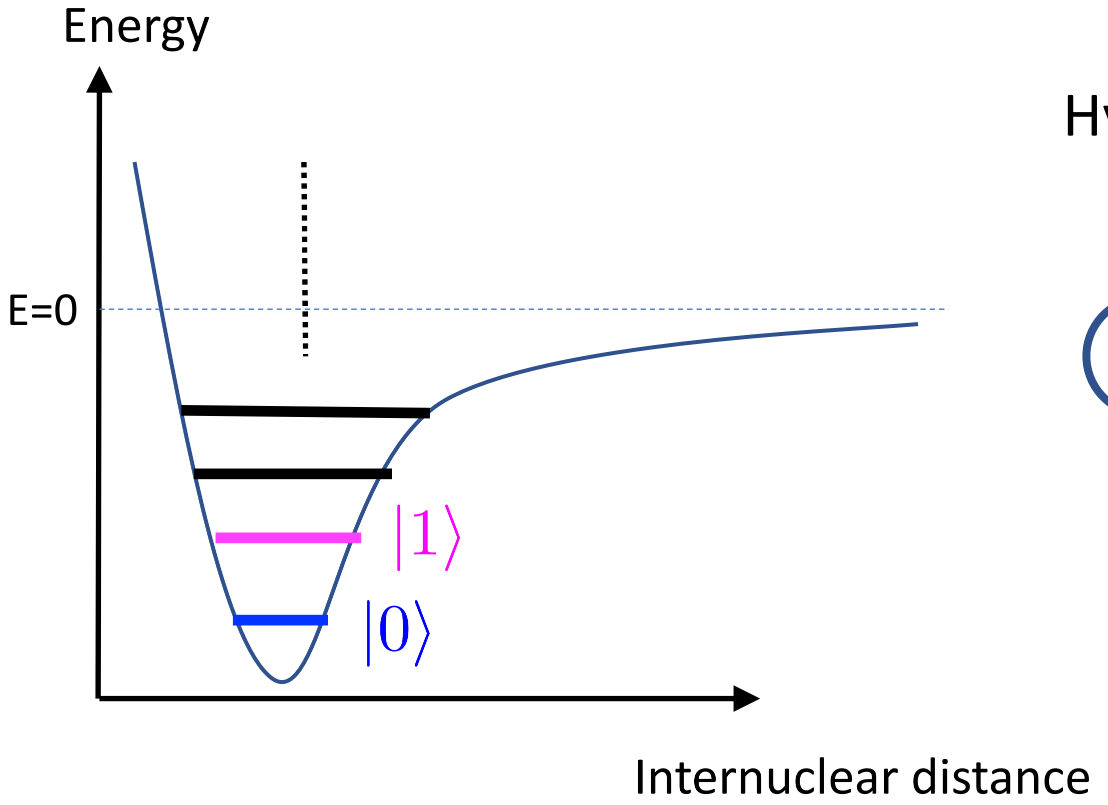
Dynamics:

$$i\frac{d}{dt}|\psi\rangle = \mathbf{H}|\psi\rangle$$

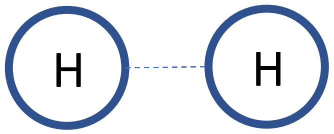
with $\mathbf{H}$ a Hermitian operator (e.g. matrix in $\mathbb{C}^{n\times n}$) : $(\mathbf{H} = \mathbf{H}^\dagger)$

PHYSICS OF INFORMATION, QUANTUM BIT (QUBIT)

$$\left(-\frac{\hbar^2}{2m}\Delta + U(x)\right)\psi_k(x) = E_k\psi_k(x), \qquad |0\rangle = \psi_0(x),\ |1\rangle = \psi_1(x)$$



Hydrogen molecule



Qubit state: $c_0|0\rangle + c_1|1\rangle \in \text{span}\{\psi_0, \psi_1\}$

QUANTUM versus CLASSICAL

Feature 2: Entanglement and tensor product for composite systems $S_1, S_2, \cdots, S_n$

Hilbert space: $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \cdots \mathcal{H}_n$ instead of $\mathcal{H}_1 \times \mathcal{H}_2 \cdots \times \mathcal{H}_n$

Dimension: $D = d_1 \times d_2 \cdots \times d_n$ instead of $d_1 + d_2 \cdots + d_n$

Entanglement and its surprising properties: $|\psi_1\rangle \otimes |\psi_2\rangle + |\widetilde{\psi}_1\rangle \otimes |\widetilde{\psi}_2\rangle \neq |\Psi\rangle \otimes |\widetilde{\Psi}\rangle$

Main source of trouble for classical simulations: huge dimensional Hilbert space

Main resource for quantum computation: huge dimensional Hilbert space

Alain Aspect, Nobel 2022

QUANTUM versus CLASSICAL

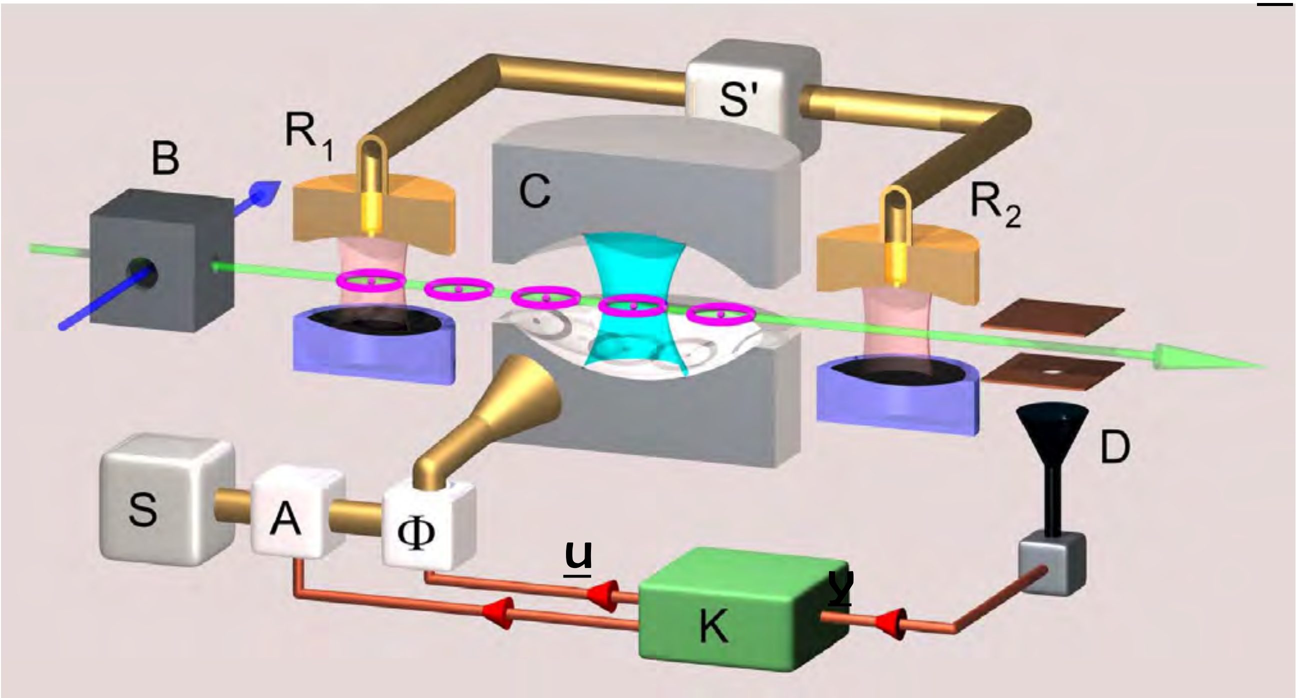
Feature 3: Randomness and irreversibility induced by measurement: any physical observable is represented by a Hermitian operator **O** with spectral decomposition $\mathbf{O} = \sum_\mu \lambda_\mu \mathbf{P}_\mu$.

Serge Haroche, Nobel 2012

- Measurement outcome λ_μ with probability $p_\mu = \langle\psi|\mathbf{P}_\mu|\psi\rangle$.
- Measurement backaction if outcome λ_μ : $|\psi_+\rangle = \frac{\mathbf{P}_\mu|\psi\rangle}{\sqrt{\langle\psi|\mathbf{P}_\mu|\psi\rangle}}$
- Puzzling for measurement and feedback control of quantum systems.
- Main resource for quantum communication and cryptography.

The first experimental realization of a quantum state feedback (2011)

The photon box of the Laboratoire Kastler-Brossel (LKB):
group of S.Haroche, J.M.Raimond and M. Brune. [16]



Stabilization of a quantum state with exactly $n = 0, 1, 2, 3, \ldots$ photon(s).

Experiment: C. Sayrin et. al., Nature 477, 73-77, September 2011.

Theory: I. Dotsenko et al., Physical Review A, 80: 013805-013813, 2009.

R. Somaraju et al., Rev. Math. Phys., 25, 1350001, 2013.

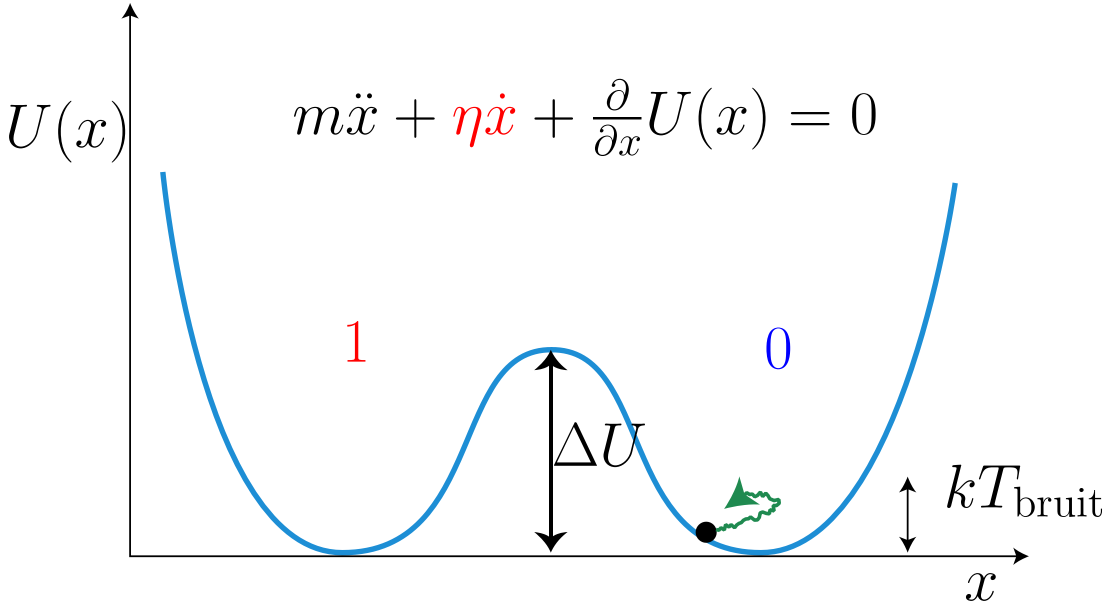
H. Amini et. al., Automatica, 49 (9): 2683-2692, 2013.

[16] Courtesy of Igor Dotsenko. Sampling period 80 μs.

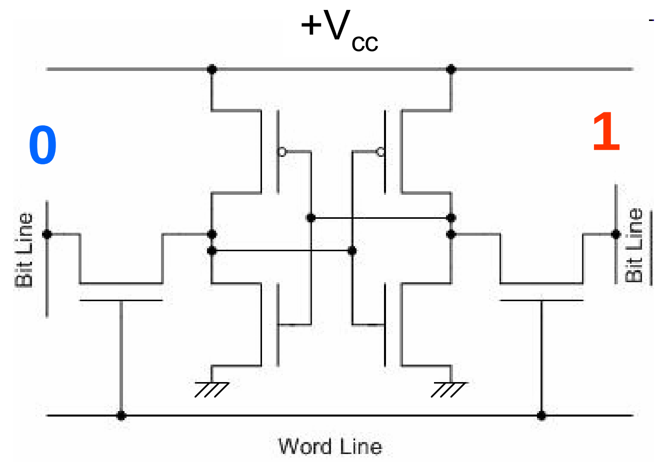
PHYSICS OF INFORMATION, CLASSICAL BIT

Classical bit: strongly dissipative bistable system

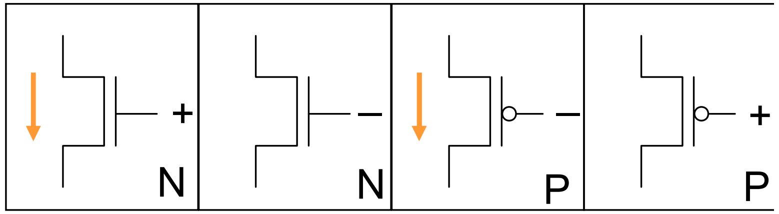
$$m\ddot{x} + \eta\dot{x} + \frac{\partial}{\partial x}U(x) = 0$$



Typical SRAM cell



CMOS Transistors:

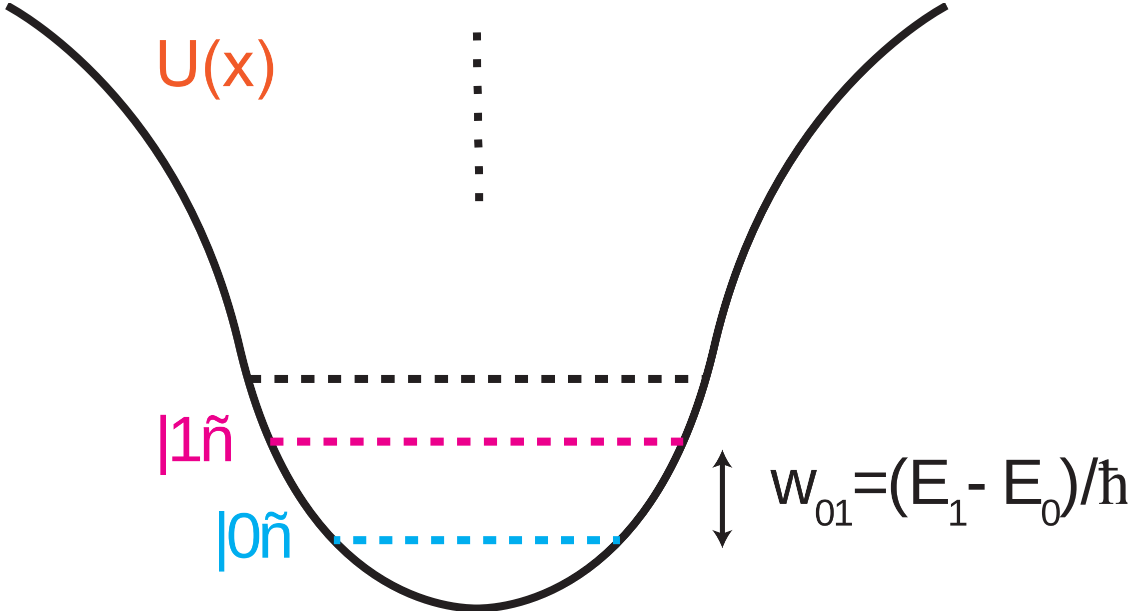


Classical bit in state 0 or 1

- Strong dissipation (friction);
- $k_B T_{noise} << \Delta U$;

BIT QUANTIQUE: RÉALISATION PHYSIQUE

$$\left(-\frac{\hbar^2}{2m}\Delta + U(x)\right)\psi_k(x) = E_k\psi_k(x), \qquad |0\rangle = \psi_0(x),\ |1\rangle = \psi_1(x)$$

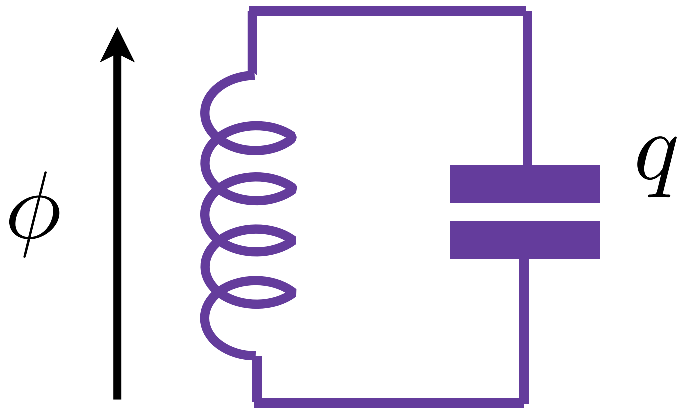


État quantique: $\left(c_0|0\rangle + c_1|1\rangle\right) \in \mathcal{H} = \mathrm{span}\left\{|0\rangle, |1\rangle\right\}$

BIT QUANTIQUE: RÉALISATION PHYSIQUE

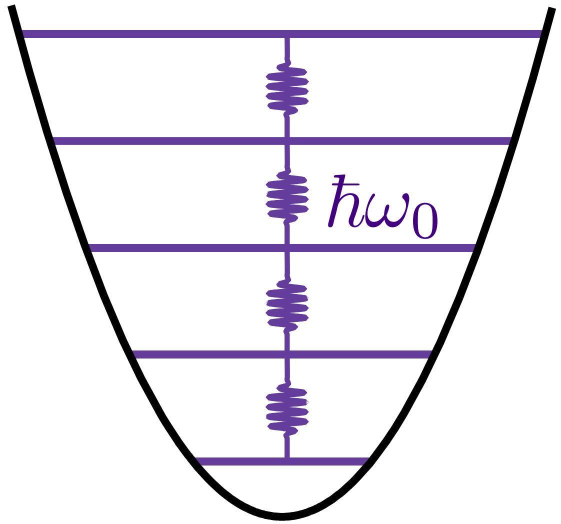
Circuits supraconducteurs

Circuit LC sans-dissipation



$$\omega_0 = 1/\sqrt{LC}$$

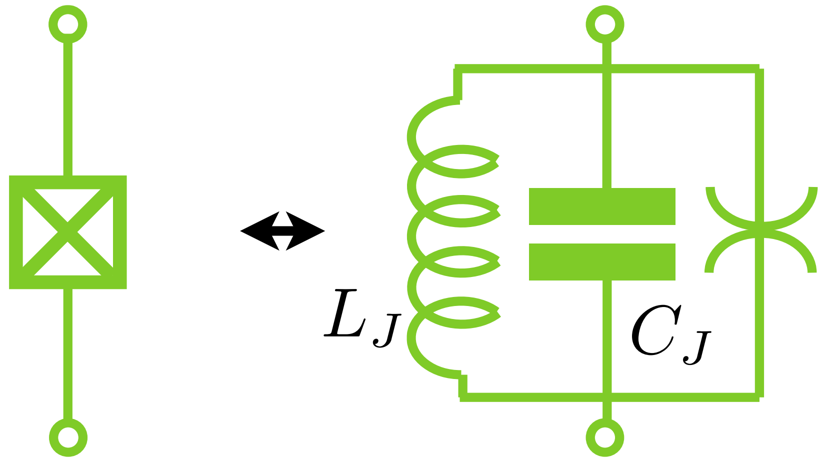
Description quantique

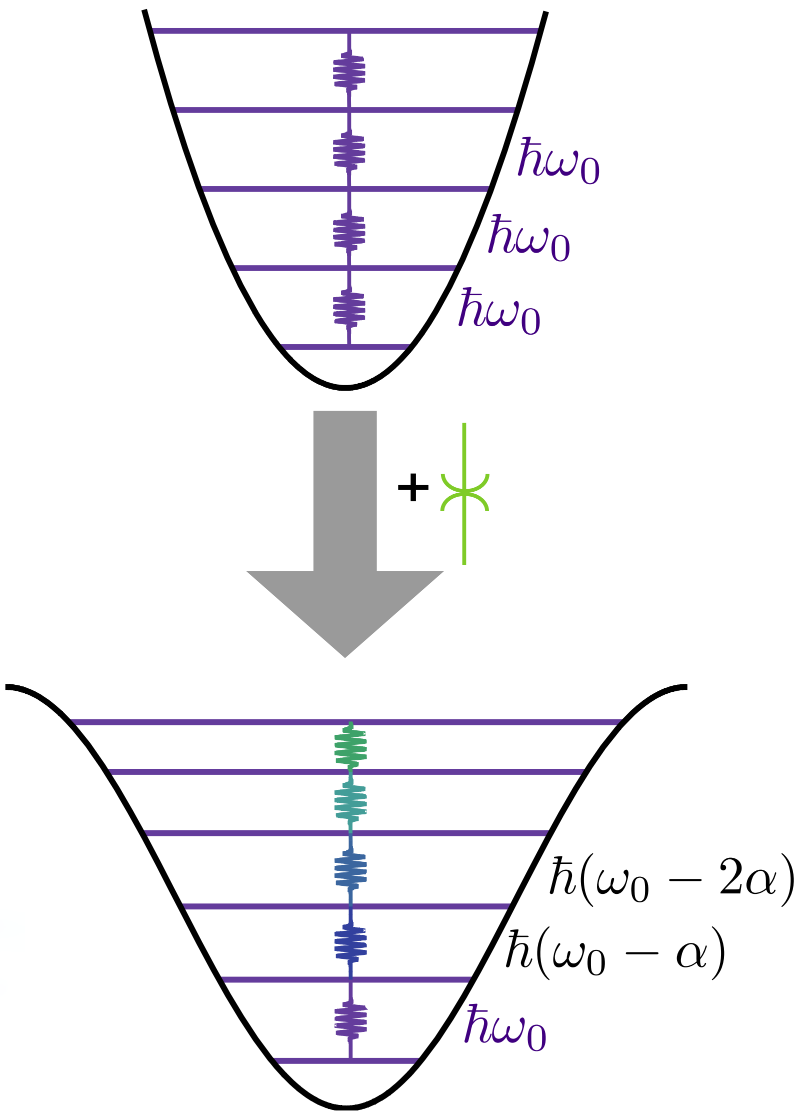


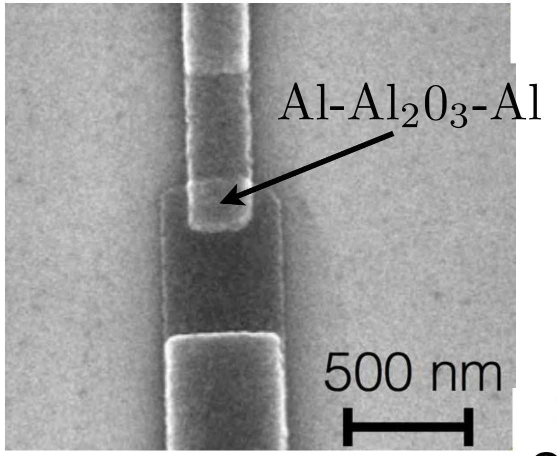
$$\left(-\frac{\hbar^2}{2C}\frac{\partial^2}{\partial\phi^2}+\frac{\phi^2}{2L}\right)\psi_k(x)=E_k\psi_k(x), \quad \mathrm{E}_k=\hbar k\omega_0$$

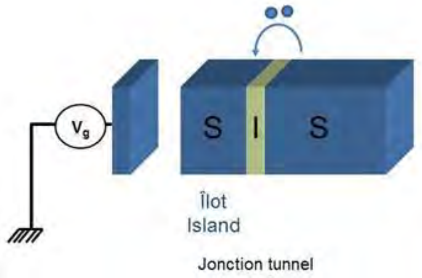
BIT QUANTIQUE: RÉALISATION PHYSIQUE

Circuit LC non-linéaire non-dissipatif





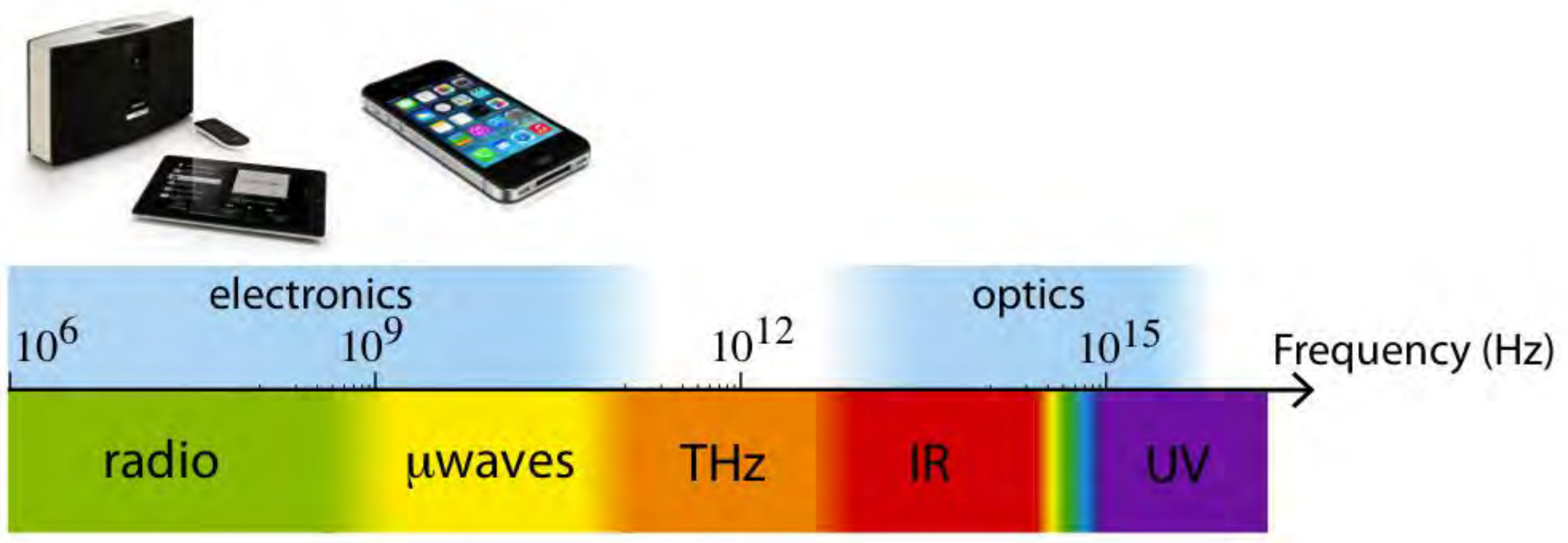




Qubit: Jonction de Josephson

BIT QUANTIQUE: RÉALISATION PHYSIQUE

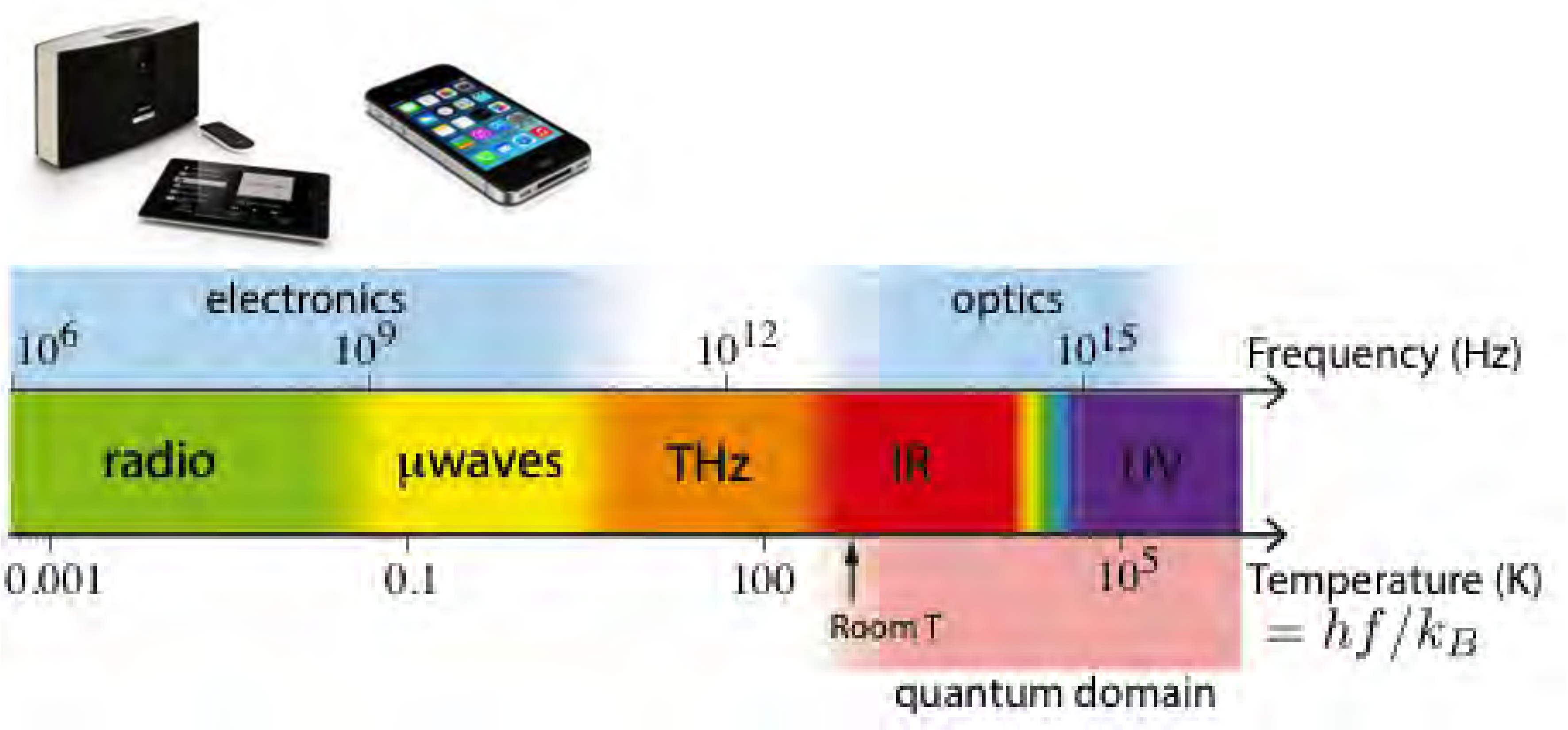
Circuits micro-ondes: comment atteindre le régime quantique?



Courtoisie de B. Huard

BIT QUANTIQUE: RÉALISATION PHYSIQUE

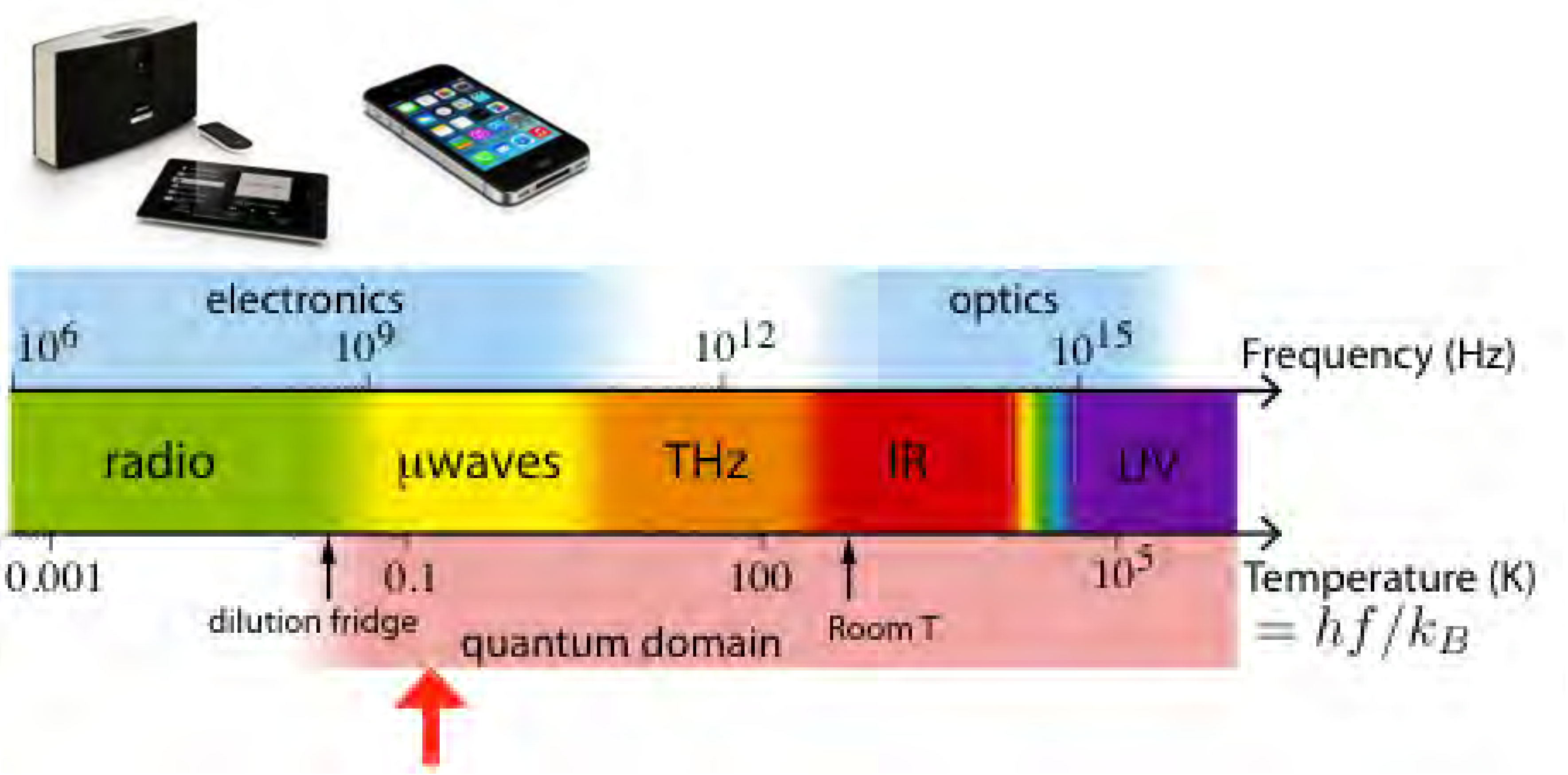
Circuits micro-ondes: comment atteindre le régime quantique?



Courtoisie de B. Huard

BIT QUANTIQUE: RÉALISATION PHYSIQUE

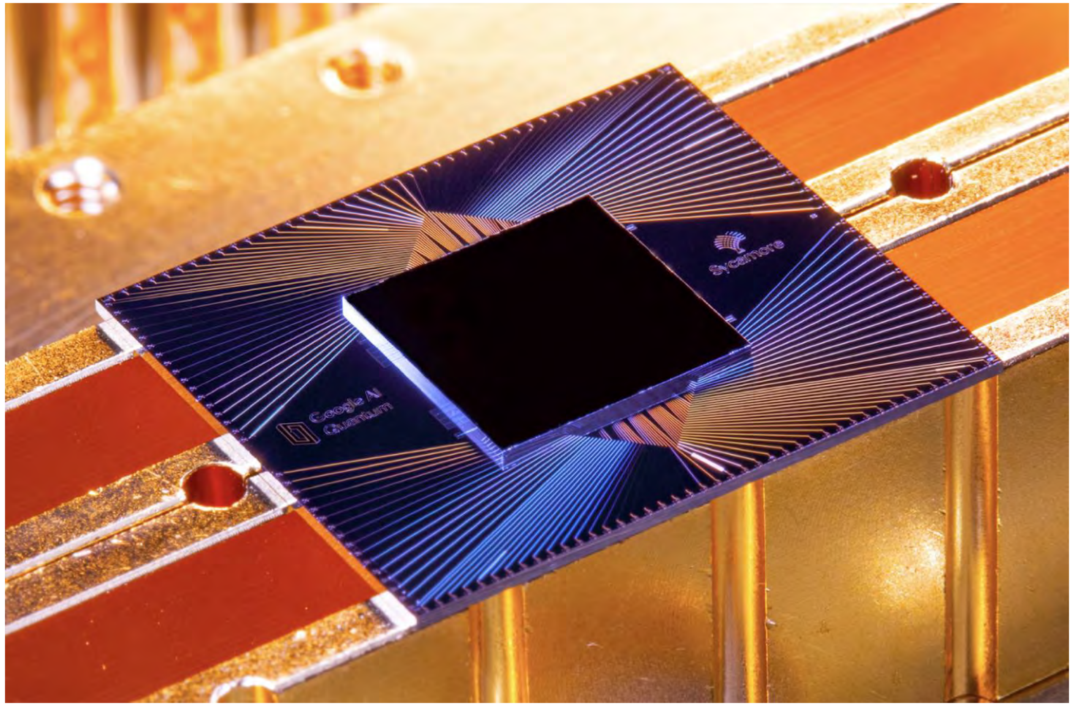
Circuits micro-ondes: comment atteindre le régime quantique?



Courtoisie de B. Huard

QUANTUM BITS ARE TOO NOISY….

Classical RAM (Random Access Memory)
~ 10^{-25} errors per bit per operation

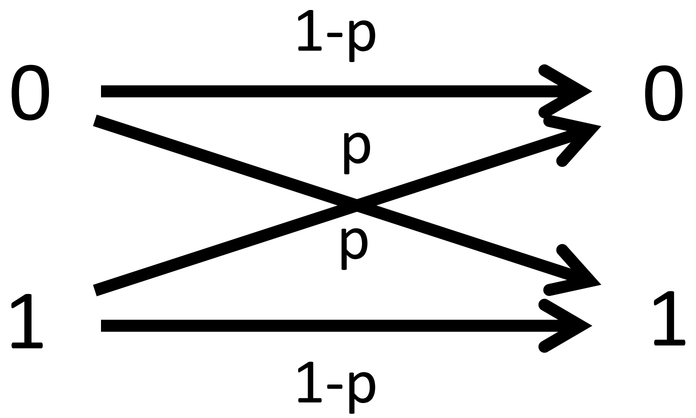


Quantum processor
~ 10^{-3} – 10^{-4} errors per bit per operation

Useful large scale quantum computation requires ~ 10^{-10} – 10^{-15}

SOLUTION: ERROR CORRECTION

Classical case: bit-flip errors



Basics of **classical** error correction: redundancy

0 → 0 0 0
1 → 1 1 1

replaced by « repetition code »

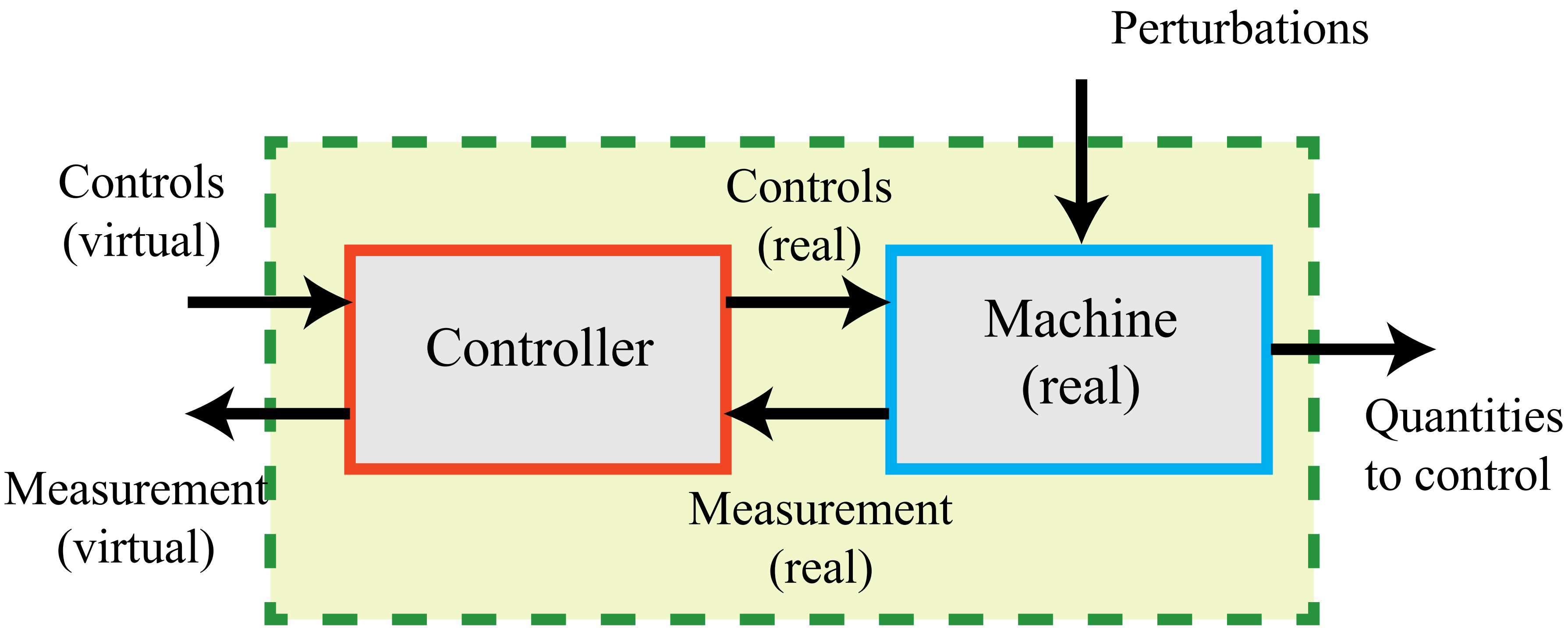
1-bit errors tractable by **majority vote**:

1 0 0 | 0 1 0 | 0 0 1
0 1 1 | 1 0 1 | 1 1 0

Probability of incorrectible 2-bit errors: $\propto p^2$ (*p* error probability per unit time)

A control problem: we measure the physical system and based on the result apply corrections

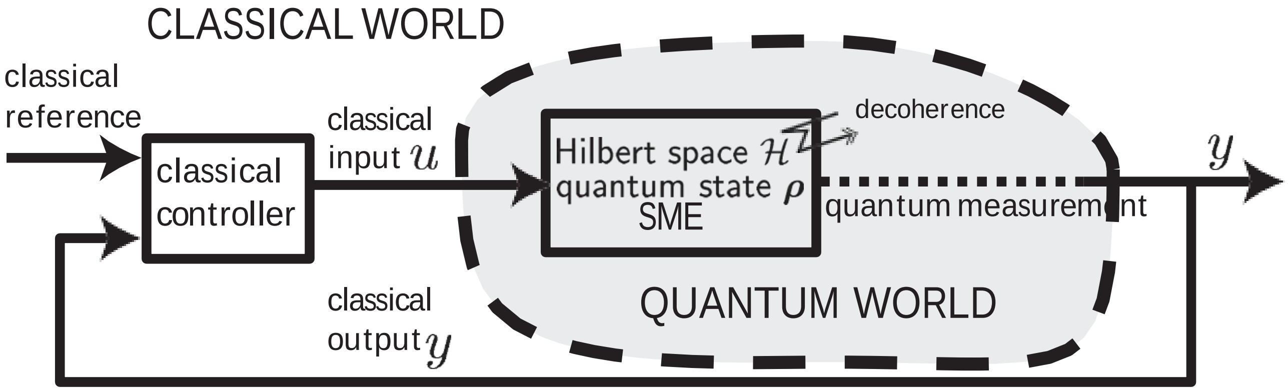
ERROR CORRECTION: A CONTROL PROBLEM



CONTROL PROBLEM: EVERYWHERE

Anti-lock Braking System

Measurement-based feedback



- P-controller (Markovian feedback[9]) for $u_t \, dt = k \, dy_t$, the ensemble average closed-loop dynamics of ρ remains governed by a linear Lindblad master equation.
- PID controller: no Lindblad master equation in closed-loop for dynamics output feedback
- Nonlinear hidden-state stochastic systems: Lyapunov state-feedback[10]; many open issues on convergence rates, delays, robustness, ...
- Short sampling times limit feedback complexity

[9] H. Wiseman, G. Milburn (2009). Quantum Measurement and Control. Cambridge University Press.

[10] See e.g.: C. Ahn et. al (2002): Continuous quantum error correction via quantum feedback control. Phys. Rev. A 65;
M. Mirrahimi, R. Handel (2007): Stabilizing feedback controls for quantum systems. SIAM Journal on Control and Optimization, 46(2), 445-467;
W. Liang, Weichao, N. Amini and P. Mason (2019): On Exponential Stabilization of N-Level Quantum Angular Momentum Systems. SIAM Journal on Control and Optimization 57(6):3939-3960.

QUANTUM ERROR CORRECTION

Much more complex in quantum case:

1- Majority vote is not appropriate as it erases precious quantum information

2- Correcting two types of errors:

- Bit-flips: $|0\rangle \longleftrightarrow |1\rangle$
- Phase-flips: $|0\rangle + |1\rangle \longleftrightarrow |0\rangle - |1\rangle$

As a result:

1) Hardware complexity and scalibility: many 100s of qubits to encode a single protected logical qubit;

2) Time-scale: short coherence times (100 microseconds for best superconducting qubits) require fast electronics for real-time error correction.

P. Shor, Phys. Rev. A 52, 1995.
A. Fowler et al., Phys. Rev. A 86, 2012.

CORRECTION D'ERREUR CLASSIQUE VS QUANTIQUE

La base de la correction d'erreur classique: redondance

0, 1 remplacé par 0 0 0, 1 1 1

Erreur à 1-bit tractable par **vote de majorité**: 1 0 0, 0 1 0, 0 0 1, 0 1 1, 1 0 1, 1 1 0

Probabilité d'erreurs à 2-bit non-corrigées: $3\varepsilon^2$ (ε prob. D'erreur par l'unité de temps)

Correction d'erreurs quantiques: erreurs de flip de bit

c_0 0 $+c_1$ 1 ⟷ c_0 0 0 0 $+c_1$ 1 1 1

Erreurs à 1 qubit tractable par **mesure de parité**: Z_1Z_2 and Z_2Z_3

CORRECTION D’ERREURS QUANTIQUES

Quatre canaux d’erreurs possibles pour chaque qubit:

c_0 0 $+c_1$ 1

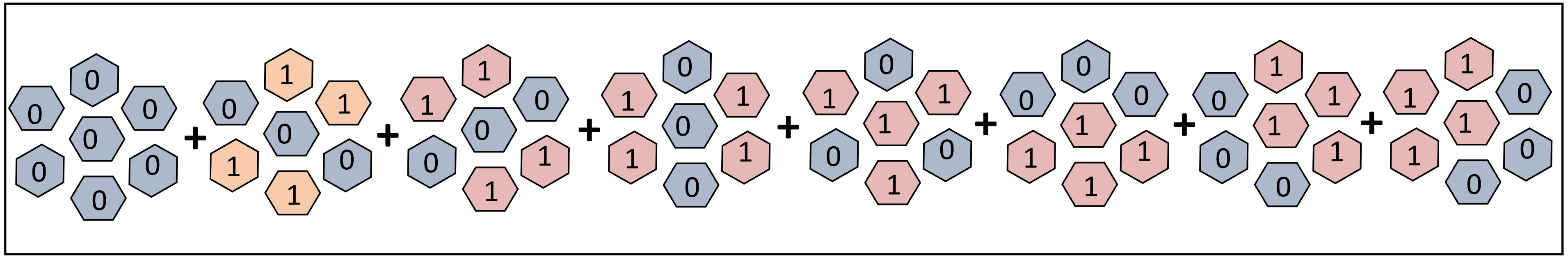
→

c_0 0 $+c_1$ 1 or c_0 1 $+c_1$ 0 or c_0 0 $- c_1$ 1 or c_0 1 $- c_1$ 0

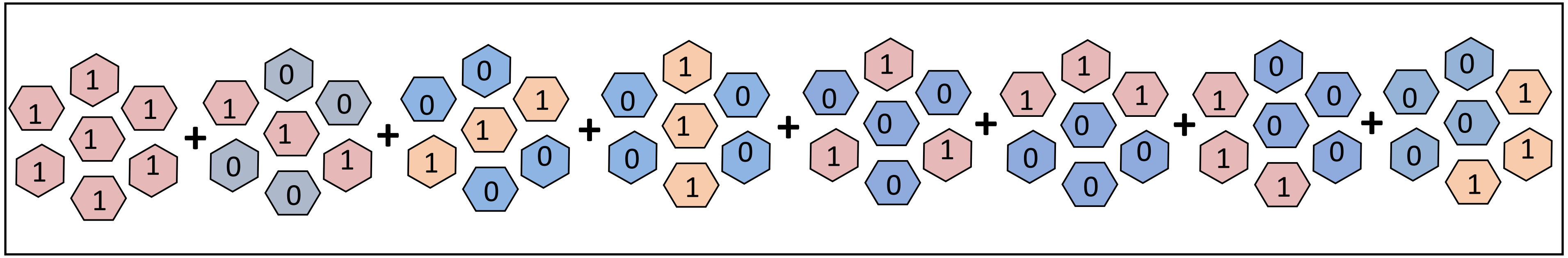
Au moins cinq qubits pour corriger toutes ces erreurs

Code de Steane à 7-qubits:

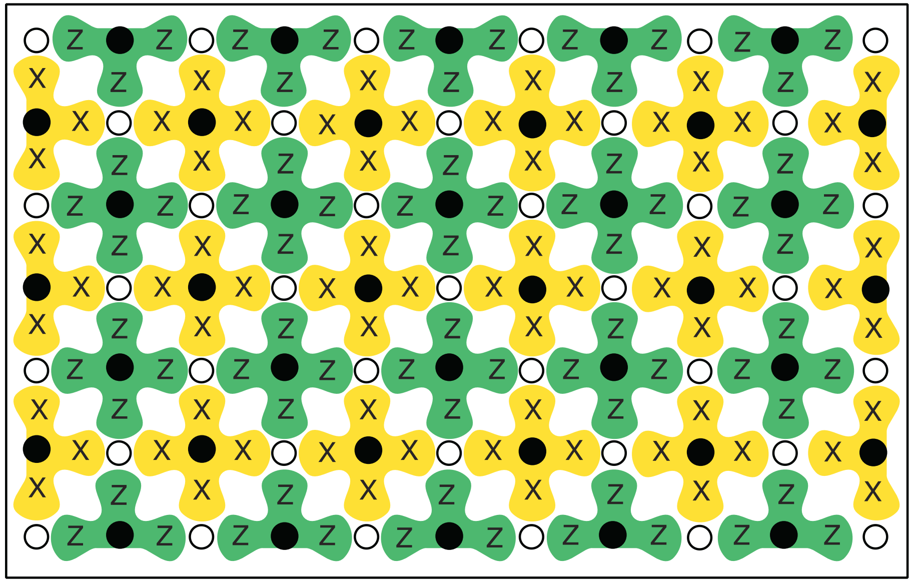
0 →



1 →



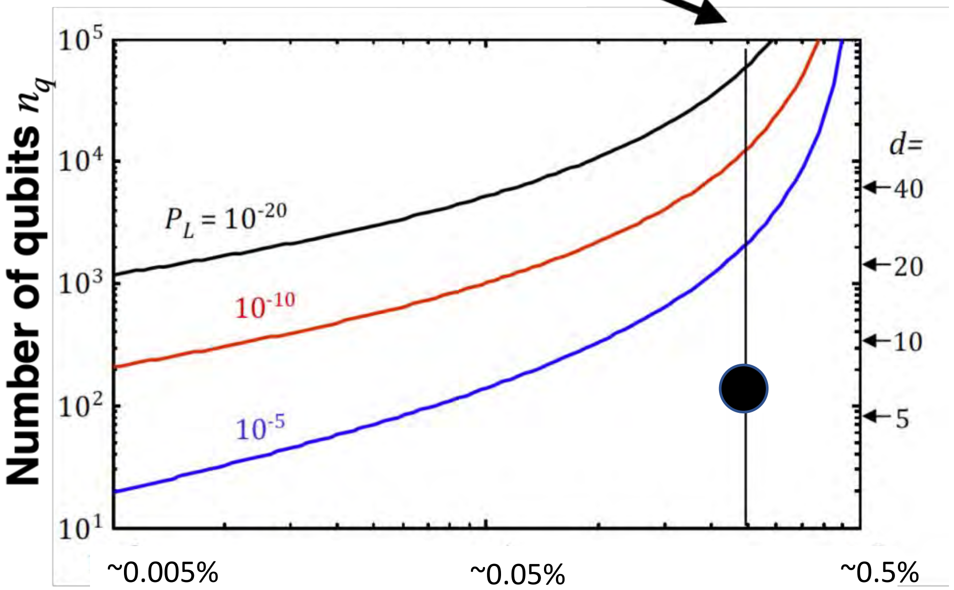
SOLUTION: QUANTUM ERROR CORRECTION



Delocalize quantum information of one logical qubit on many physical qubits for more robustness

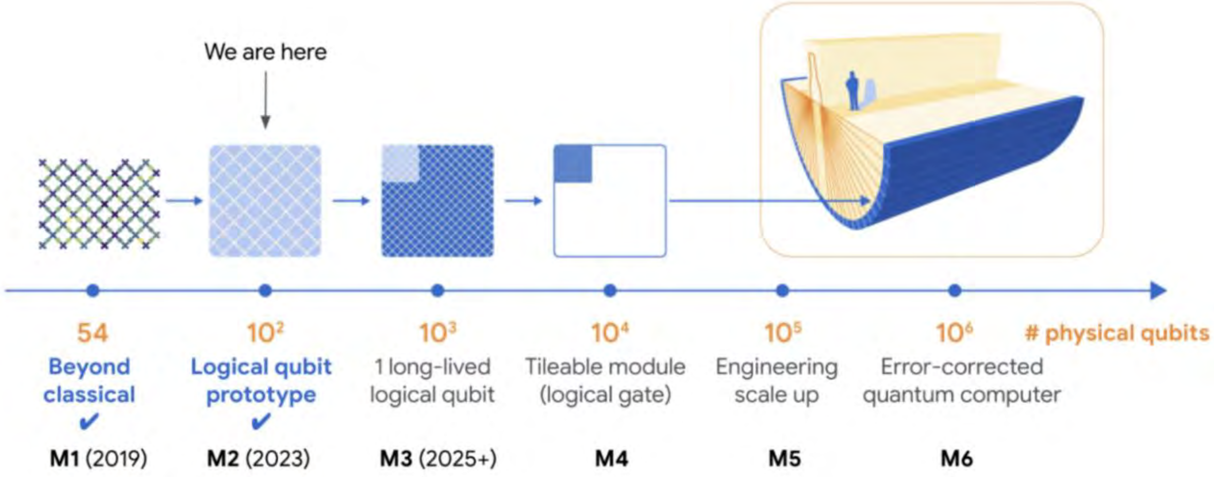
A. Fowler et al., Phys. Rev. A 86, 2012.

Google Quantum AI, Nature, 2025



Physical error probability per bit per operation

Peter Shor



Google roadmap

IDEA: BACK TO EARLY DAYS OF CONTROL

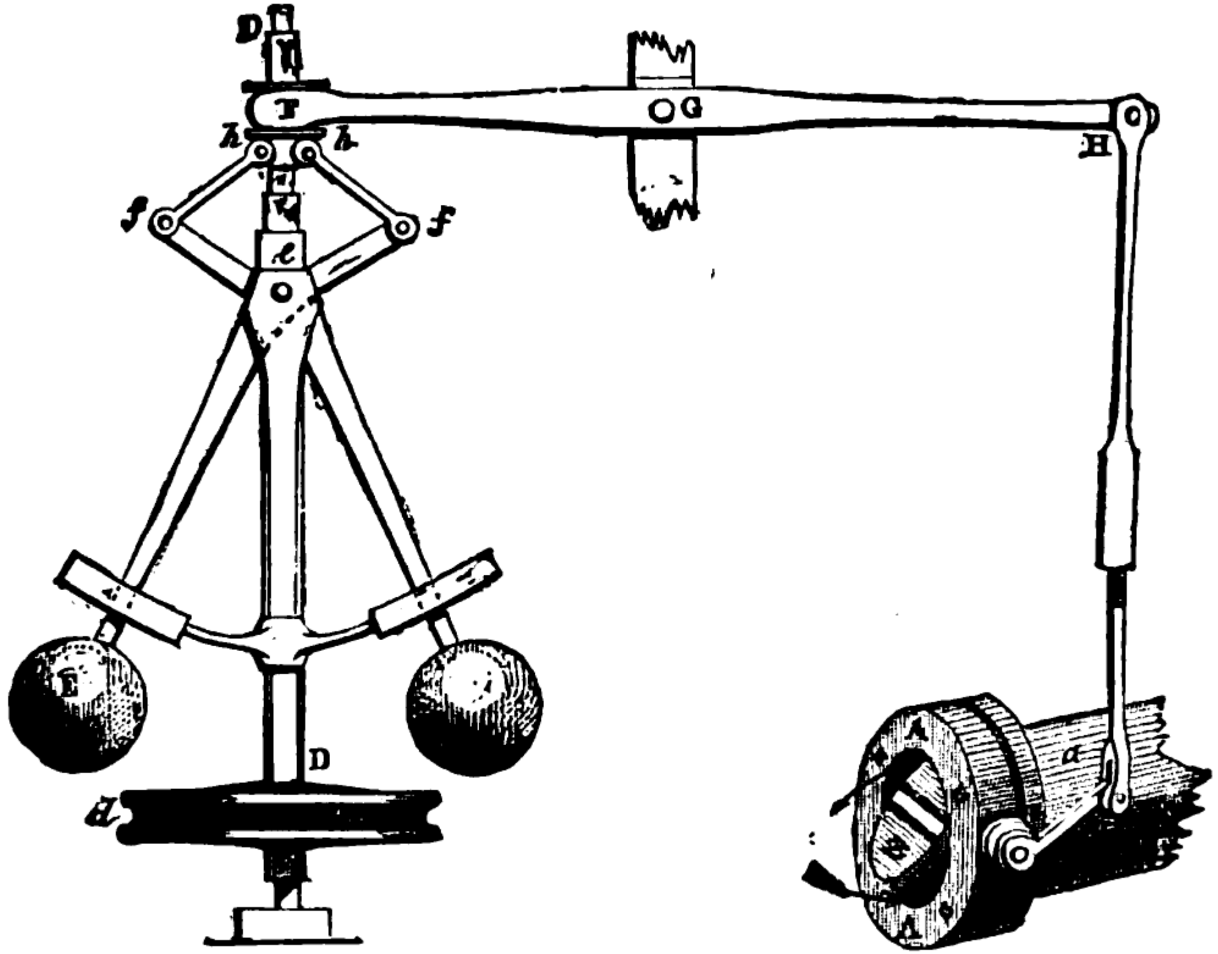


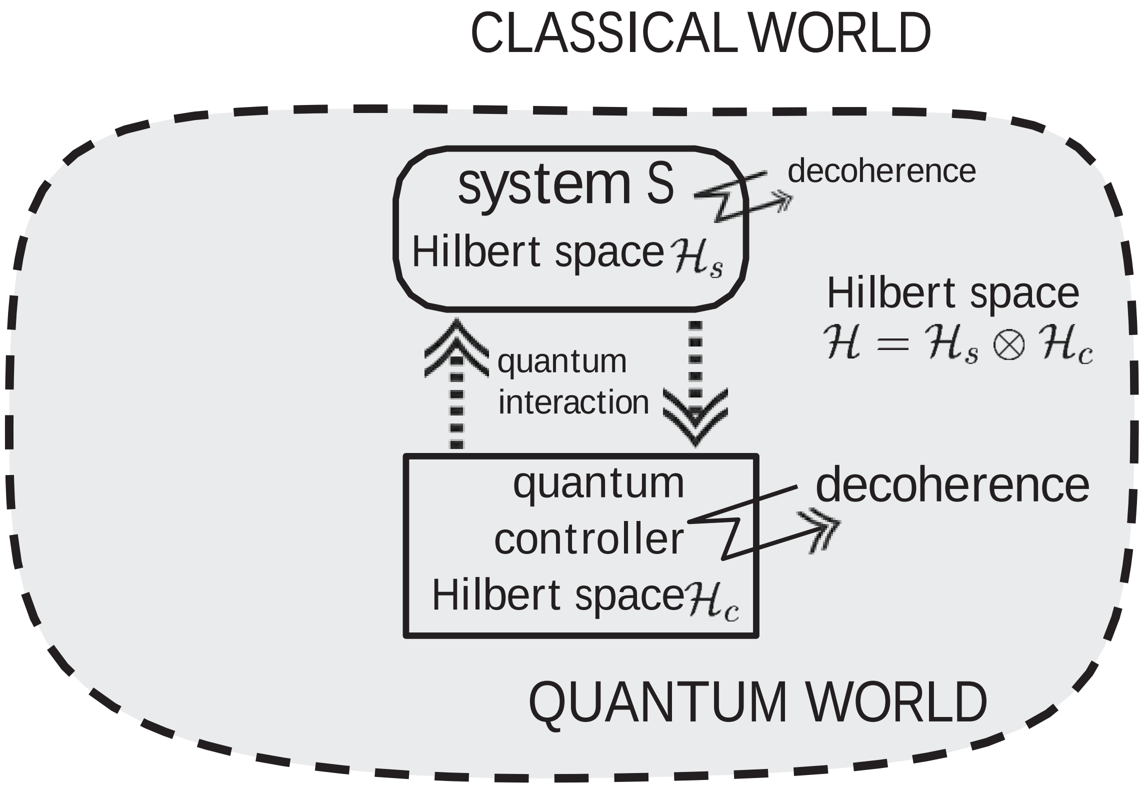
FIG. 4.—*Governor and Throttle-Valve.*

Centrifugial Watt regulator for steam engines

J.C Maxwell, On governors, 1868.

The dissipation (friction) of the governor should be strong enough to ensure the stability!

Quantum analogue of Watt speed governor: a dissipative mechanical system controls another mechanical system [12]



Optical pumping (Kastler 1950), coherent population trapping (Arimondo 1996)

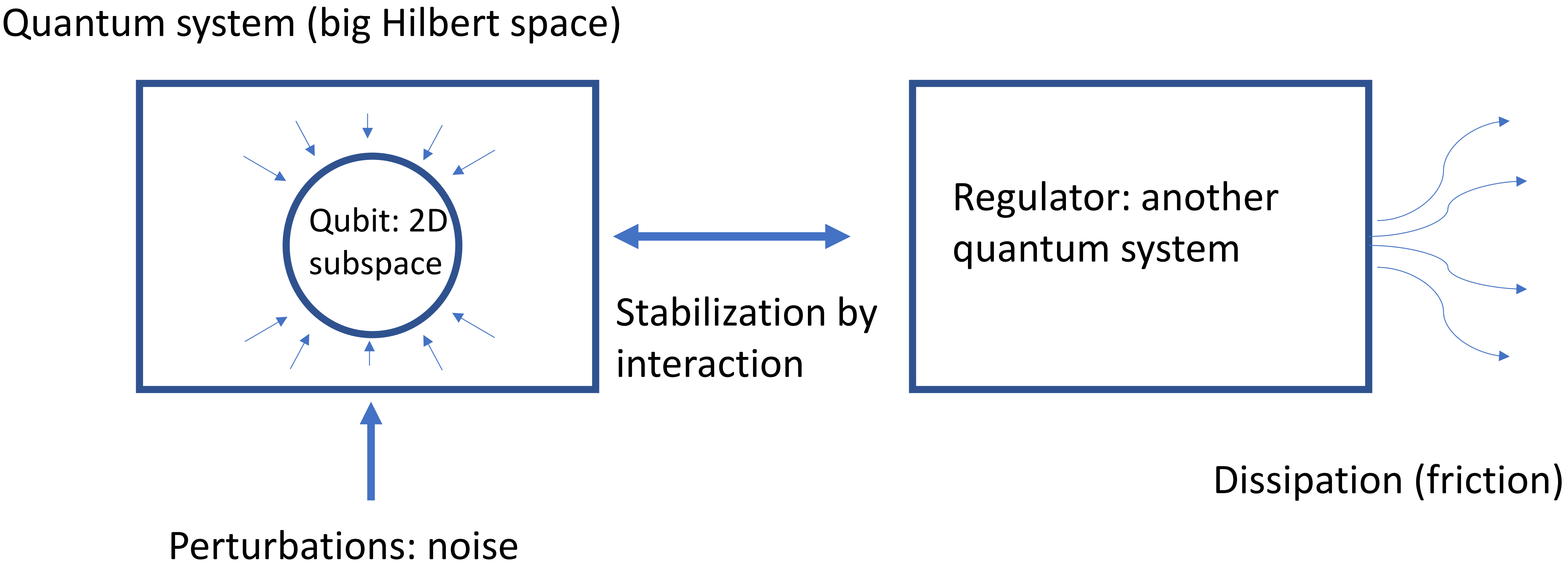
Dissipation engineering, autonomous feedback: (Zoller, Cirac, Wolf, Verstraete, Devoret, Schoelkopf, Siddiqi, Martinis, M!2lmer, Raimond, Brune,. . . , Lloyd, Viola, Ticozzi, Leghtas, Mirrahimi, Sarlette, PR, ...)

(S,L,H) theory and linear quantum systems: quantum feedback networks based on stochastic Schrëdinger equation, Heisenberg picture (Gardiner, Yurke, Mabuchi, Genoni, Serafini, Milburn, Wiseman, Doherty, . . . , Gough, James, Petersen, Nurdin, Yamamoto, Zhang, Dong, . . .)

Stability analysis: Kraus maps and Lindblad propagators are always contractions (non commutative diffusion and consensus).

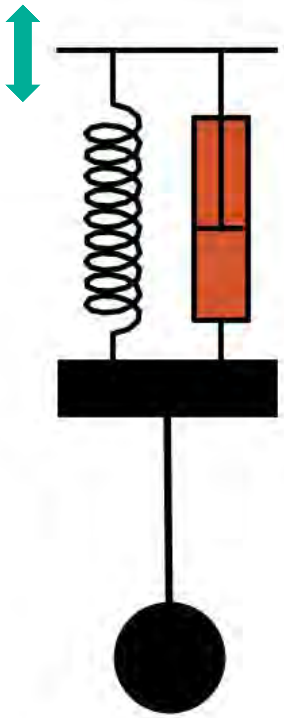
[12] J.C. Maxwell (1868): On governors. Proc. of the Royal Society, No.100.

IDEA: A QUANTUM REGULATOR



Stabilization of the 2D subspace is ensured without perturbing the dynamics inside the subspace

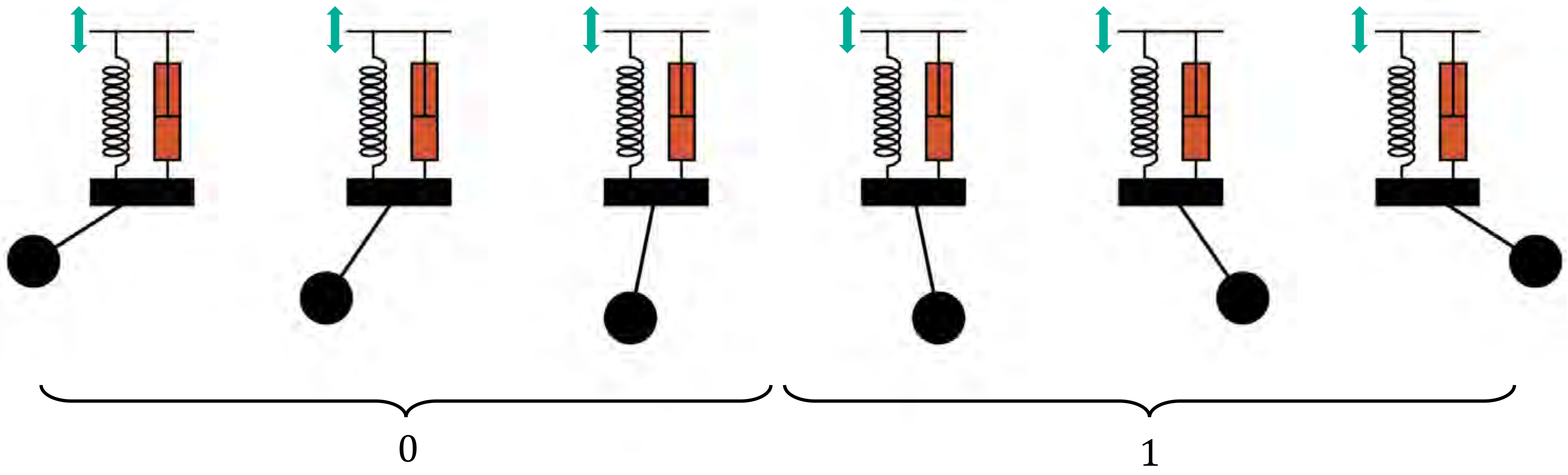
MAIN IDEA IN A CLASSICAL PICTURE



Driven damped oscillator
coupled to a pendulum.

Courtesy of Raphaël Lescanne

A BI-STABLE SYSTEM

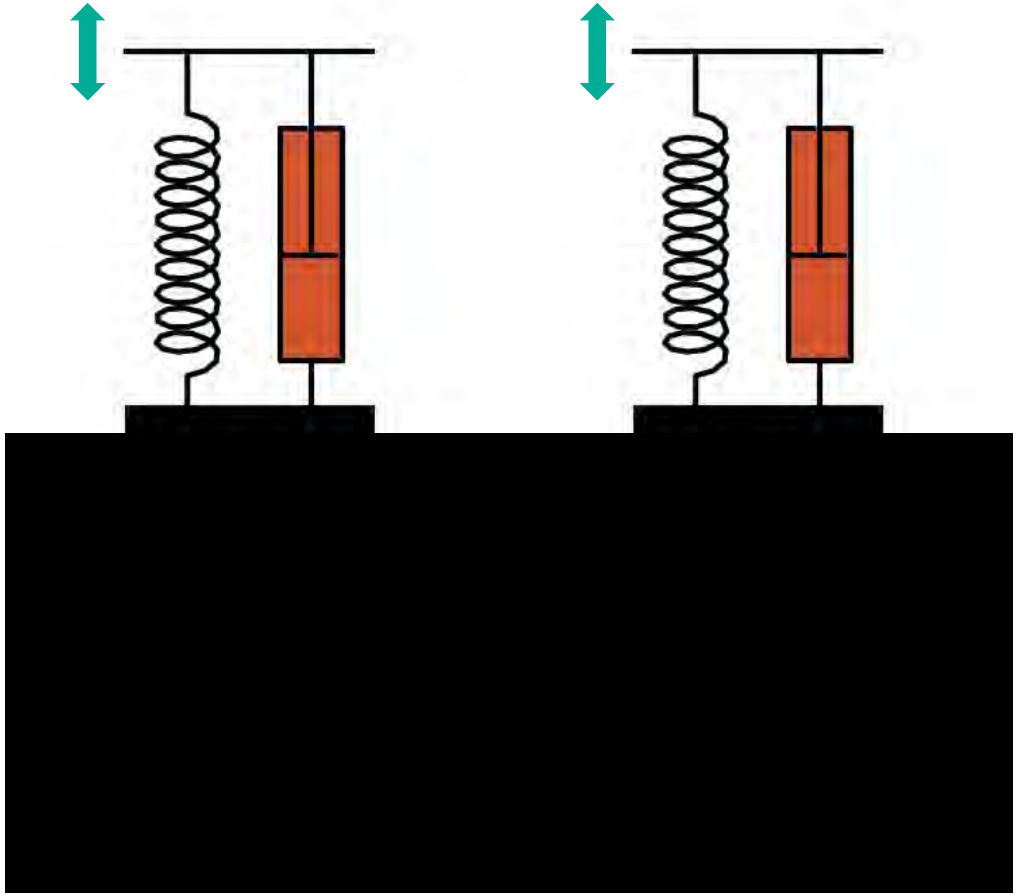


There are **2 steady states** in which we can encode information

Courtesy of Raphaël Lescanne

MAIN IDEA IN A CLASSICAL PICTURE

Stabilization regardless of the state



Neither the **drive** nor the **dissipation** can **distinguish** between 0 and 1

Important to preserve quantum coherence

Courtesy of Raphaël Lescanne

9

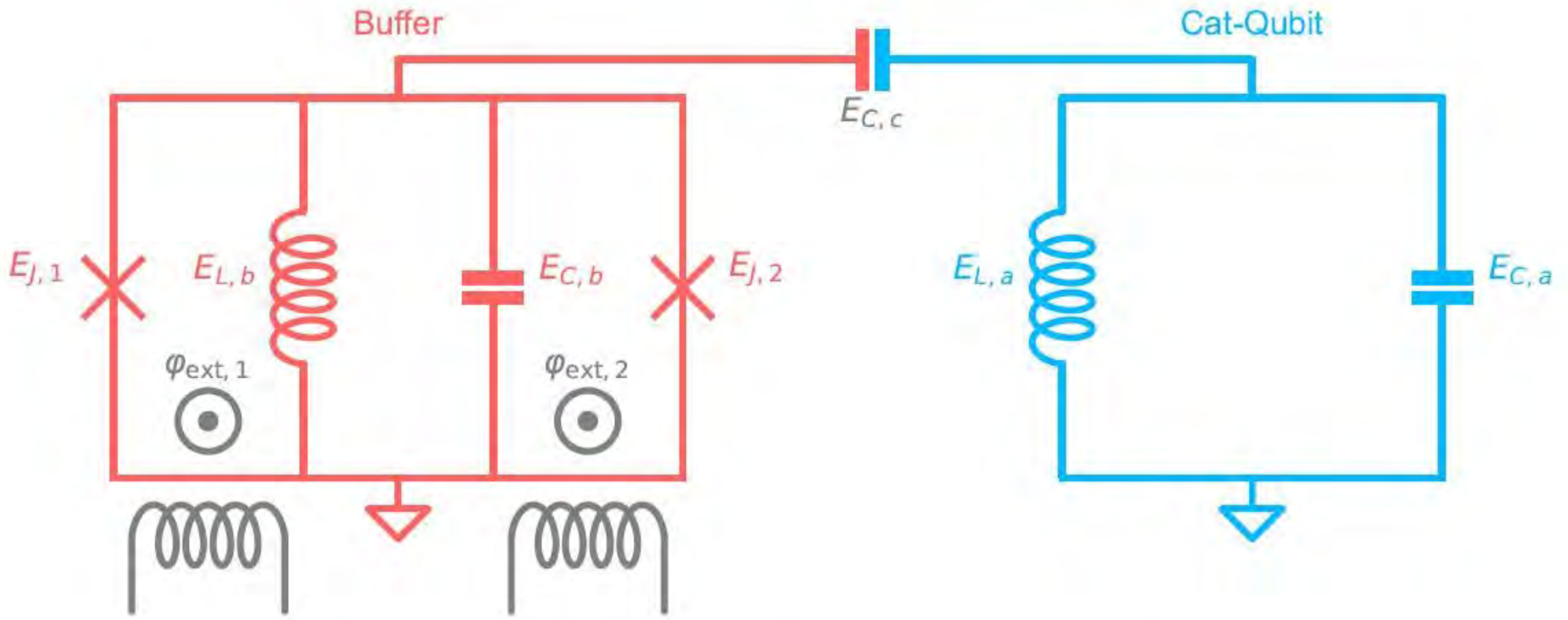
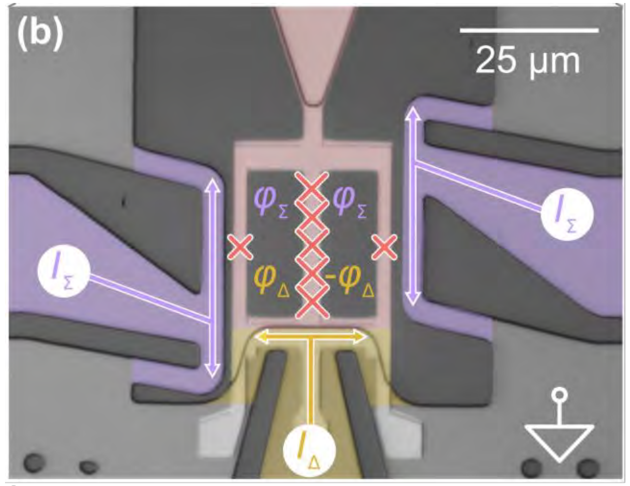
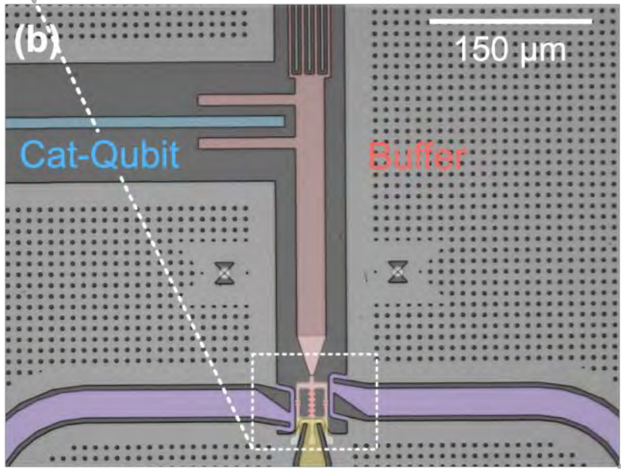


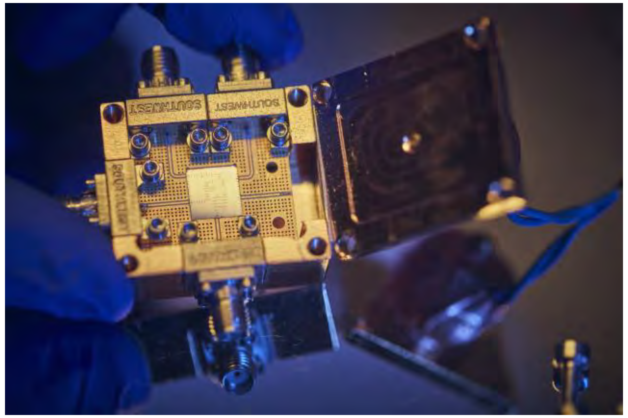
Figure S3. Equivalent circuit diagram. The cat-qubit (blue), a linear resonator, is capacitively coupled to the buffer (red). One recovers the circuit of Fig. 2 by replacing the buffer inductance with a 5-junction array and by setting $\varphi_\Sigma = (\varphi_{\text{ext},1} + \varphi_{\text{ext},2})/2$ and $\varphi_\Delta = (\varphi_{\text{ext},1} - \varphi_{\text{ext},2})/2$. Not shown here: the buffer is capacitively coupled to a transmission line, the cat-qubit resonator is coupled to a transmon qubit

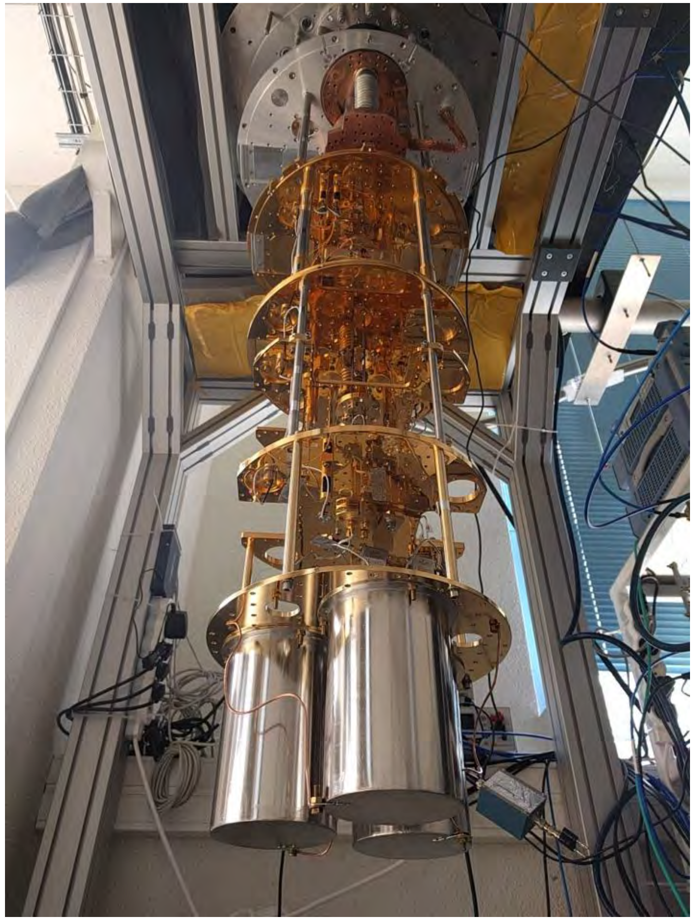
[23] R. Lescanne, M. Villiers, Th. Peronnin, . . . , M. Mirrahimi and Z. Leghtas: Exponential suppression of bit-flips in a qubit encoded in an oscillator. 2020, Nature Physics

QUANTUM VERSION: SUPERCONDCUTING CAT-QUBIT









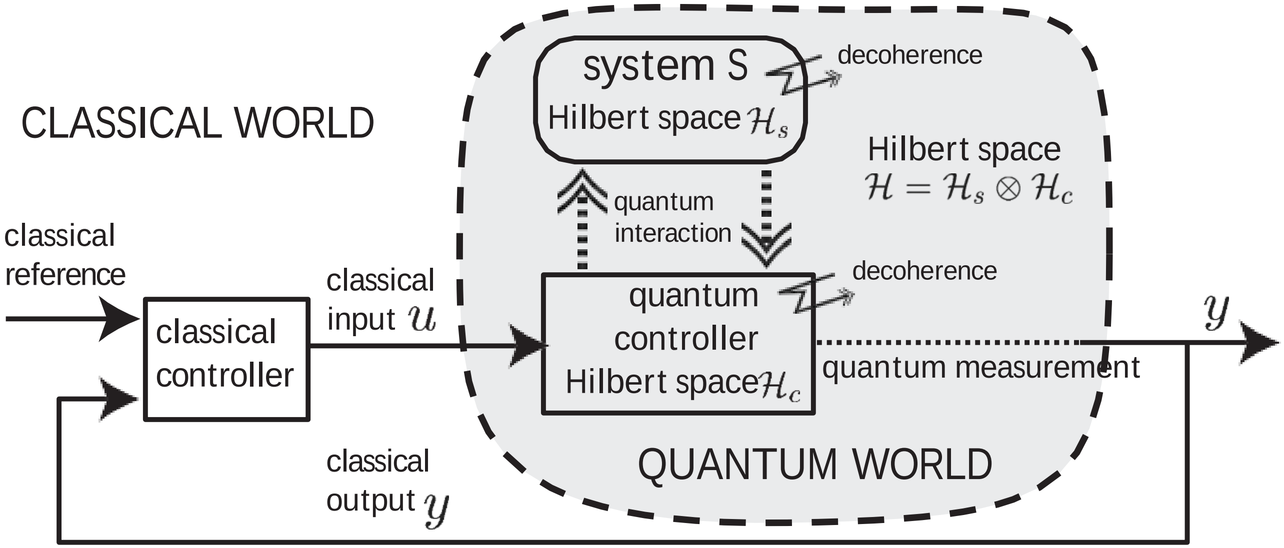
Dilution cryostat : temperature 20mK

Cat-qubits: roadmap pursued by Alice&Bob (France), Amazon (USA), and some academic groups

A NEW DISCIPLINE: QUANTUM ENGINEERING

- Physics: experimental and theoretical
- Applied mathematics: Partial differential equations, Probability theory, Dynamical systems and control
- Computer science: software aspects

Quantum feedback engineering for robust quantum information processing



To protect quantum information stored in system S:

- fast stabilization and protection mainly achieved by quantum controllers (autonomous feedback stabilizing decoherence-free sub-spaces);
- slow decoherence and perturbations, parameter estimation mainly tackled by classical controllers and estimation algorithms (measurement-based feedback and estimation 11inishing the job11)

Need of adapted mathematical and numerical methods for high-precision dynamical modeling and control based on (stochastic) master equations.